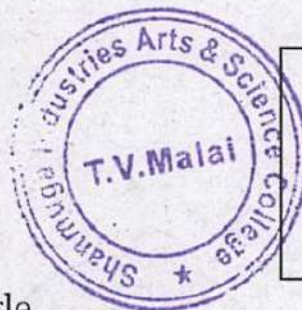


SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Fejer's theorem.
17. Let f and D_2f be continuous on a rectangle $[a, b] \times [c, d]$. Let p & q be differentiable on $[c, d]$, where $p(y) \in [a, b]$ and $q(y) \in [a, b]$ for each y in $[c, d]$. Define $F(y) = \int_{p(y)}^{q(y)} f(x, y) dx$, if $y \in [c, d]$. Prove that $F'(y)$ exists for each y in (c, d) and is given

$$F'(y) = \int_{p(y)}^{q(y)} D_2 f(x, y) dx + f(q(y), y)q'(y) - f(p(y), y)p'(y)$$
18. State and prove implicit function theorem.
19. (a) If $E_1, E_2, \dots$ are any subsets of $[a, b]$, show that $\overline{m}\left(\bigcup_{n=1}^{\infty} E_n\right) \leq \sum_{n=1}^{\infty} \overline{m} E_n$.
 (b) If $E_1, E_2, \dots$ are pair wise disjoint subsets of $[a, b]$, show that $\underline{m}\left(\bigcup_{n=1}^{\infty} E_n\right) \geq \sum_{n=1}^{\infty} \underline{m} E_n$.
20. Explain the Fatou's Lemma.



APRIL/MAY 2025

DMA22 — REAL ANALYSIS – II

Time : Three hours

Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define orthogonal system.
2. State Jordan's test.
3. Define Linear functions.
4. Define Jacobian matrix of f at c .
5. If $f = u + iv$ is a complex-valued function with a derivative at a point z in c , prove that $J_f(z) = |f'(z)|^2$.
6. State quadratic form for real-valued function.
7. If $E \subset [a, b]$, prove that $\underline{m}E \leq \overline{m}E$.
8. Define Lebesgue integrable.

9. If $f(x) = \frac{1}{\sqrt[3]{x}}$ ($0 < x \leq 1$), find $f(0) = 0$.

10. State the Schwarz Inequality.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) State and prove Riesz-Fischer Theorem.

Or

(b) Assume that $f \in L(I)$. Show that for each real β , $\lim_{\alpha \rightarrow +\infty} \int_I f(t) \sin(\alpha t + \beta) dt = 0$.

12. (a) State and prove Cauchy-Riemann equations.

Or

(b) Let S be an open connected subset of R^n , and let $f: S \rightarrow R^m$ be differentiable at each point of S . If $f'(c) = 0$ for each c in S , show that f is constant on S .

13. (a) Let A be an open subset of R^n and assume that $f: A \rightarrow R^n$ is continuous and has finite partial derivatives $D_i f_i$ on A . If f is 1-1 on A and if $J_f(x) \neq 0$ for each x in A , prove that $f(A)$ is open.

Or

(b) Derive the second-derivative test for extrema.

14. (a) If $E \subset [a, b]$, then prove that $\overline{m}E + \underline{m}E' = b - a$ (where $E' = [a, b] - E$).

Or

(b) If f and g are functions on $[a, b]$, if $f(x) = g(x)$ almost everywhere ($a \leq x \leq b$), and if f is measurable, show that g is also measurable.

15. (a) If f is a bounded function in $\mathcal{L}[a, b]$, and if $a < c < b$, prove that $f \in \mathcal{L}[a, c]$, $f \in \mathcal{L}[c, b]$, and $\int_a^b f = \int_a^c f + \int_c^b f$.

Or

(b) State and prove Minkowski inequality.

